Riemann Sum Practice Problems

Questions:

1. Approximate the area under the curve $\Delta x = \frac{1 - -2}{6} = \frac{1}{2} = .5$

$$f(x) = x^2 + 2, \quad -2 \le x \le 1$$

with a Riemann sum, using six sub-intervals and right endpoints.

$$\frac{1}{2}\left[f(-1.5)+f(-1)+f(-.5)+f(0)+f(.5)+f(1)\right] = \frac{1}{2}\sum_{k=1}^{6}\left[\left(\tfrac{1}{2}k-2\right)^2+2\right]$$

$$= \boxed{8.375}$$

2 Approximate the area under the curve

$$f(x) = \sqrt{x+1}, \quad -1 \le x \le 0 \qquad \Delta x = \frac{0 - -1}{4} = \frac{1}{4}$$

with a Riemann sum, using 4 sub-intervals and left endpoints.

$$\frac{1}{4}\left[f(-1)+f(-.75)+f(-.5)+f(-.25)\right] = \frac{1}{4}\sum_{k=0}^{3}\sqrt{\left(-1+\tfrac{1}{4}k\right)+1}$$

$$\approx \boxed{1.167}$$

3. **a)** Approximate the value of the integral:

$$\int_1^3 x^3 - 3\,dx$$

$\Delta x = \frac{3-1}{4} = \frac{1}{2}$

a) $\frac{1}{2}\left[f(1.5)+f(2)+f(2.5)+f(3)\right] \approx \boxed{21}$

with a Riemann sum, using three sub-intervals and right endpoints.

b) using five sub-intervals and left endpoints. $\Delta x = \frac{3-1}{5} = \frac{2}{5}$

b) $\frac{2}{5}\left[f(1)+f(1.4)+f(1.8)+f(2.2)+f(2.6)\right] = \boxed{16}$

c) Compute the exact value of the integral using the ~~Fundamental Theorem of Calculus.~~ CALCULATOR

$= 14$

4. A rectangular canal, 5m wide and 100m long has an uneven bottom. Depth measurements are taken at every 20m along the length of the canal. Use these depth measurements to construct a Riemann sum using right endpoints to estimate the volume of water in the canal.

X Distance	0m	20m	40m	60m	80m	100m
Y Depth	2.0m	1.6m	1.8m	2.1m	2.1m	1.9m

$(20-0)1.6+(40-20)1.8+(60-40)2.1+(80-60)2.1+(100-80)1.9$

CALCULUS WORKSHEET ON RIEMANN SUMS

1. Water is flowing into a tank over a 12-hour period. The rate at which water is flowing into the tank at various times is measured, and the results are given in the table below, where $R(t)$ is measured in gallons per hour and t is measured in hours. The tank contains 150 gallons of water when t = 0.

t (hours)	0	5	8	12
$R(t)$ (gal/hr)	8	8.8	9.3	9.2

a) Estimate the number of gallons of water in the tank at the end of 12 hours by using a left Riemann sum with three subintervals and values from the table. Draw the rectangles that you use, and show the computations that lead to your answer.

b) Estimate the number of gallons of water in the tank at the end of 12 hours by using a right Riemann sum with three subintervals and values from the table. Draw the rectangles that you use, and show the computations that lead to your answer.

2. Oil is being pumped into a tank over a 12-hour period. The tank contains 120 gallons of oil when t = 0. The rate at which oil is flowing into the tank at various times is modeled by a differentiable function R for $0 \le t \le 12$, where t is measured in hours and $R(t)$ is measured in gallons per hours. Values of $R(t)$ at selected values of time t are shown in the table below.

t (hours)	0	3	5	9	12
$R(t)$ (gallons per hour)	8.9	6.8	6.4	5.9	5.7

a) Estimate the number of gallons of oil in the tank at t = 12 hours by using a midpoint Riemann sum with ~~three~~ two subintervals and values from the table. Show the computations that lead to your answer.

b) Estimate the number of gallons of oil in the tank at t = 12 hours by using the Trapezoidal Rule with four subintervals and values from the table. Show the computations that lead to your answer.

3. A hot cup of coffee is taken into a classroom and set on a desk to cool. When t = 0, the temperature of the coffee is 113° F. The rate at which the temperature of the coffee is dropping is modeled by a differentiable function R for $0 \le t \le 8$, where $R(t)$ is measured in degrees Fahrenheit per minute and t is measured in minutes. Values of $R(t)$ at selected values of time t are shown in the table below.

t (minutes)	0	3	5	8
$R(t)$ (° F/min.)	5.5	2.7	1.6	0.8

a) Estimate the temperature of the coffee at t = 8 minutes by using a left Riemann sum with three subintervals and values from the table. Show the computations that lead to your answer.

b) Estimate the temperature of the coffee at t = 8 minutes by using a right Riemann sum with three subintervals and values from the table. Show the computations that lead to your answer.

4. Estimate the area bounded by the curve and the x-axis on [1, 6] using the by finding:

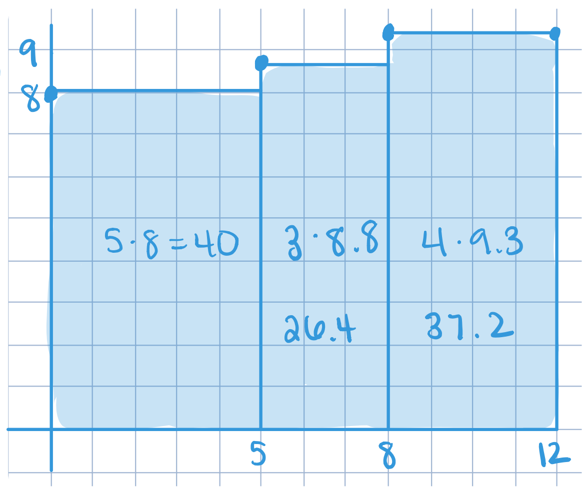
(a) a left Riemann sum with 5 equal subintervals

(b) a right Riemann sum with 5 equal subintervals

5. Estimate the area bounded by $y = 4 - x^2$, [0,2] and the x-axis on the given interval using the indicated number of subintervals by finding (a) a left Riemann sum, n=4, (b) a right Riemann sum, n = 4, (c) a midpoint Riemann Sum, n=2.

1

a

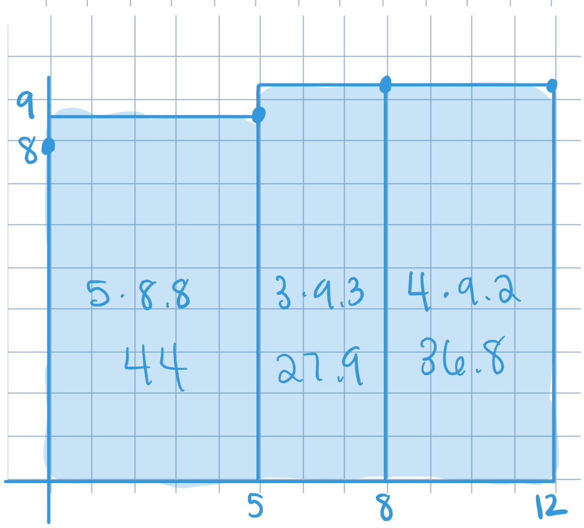


LRS

40
26.4
37.2
103.6 gallons

150 + 103.6 = 253.6 gallons

b



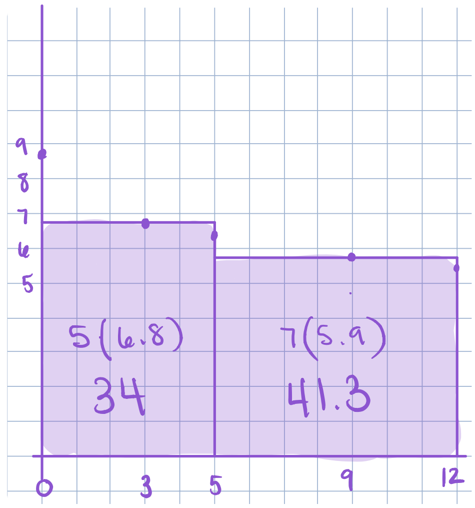
RRS

44
27.9
36.8
108.7 gallons

150 + 108.7 = 258.7 gallons

2

a

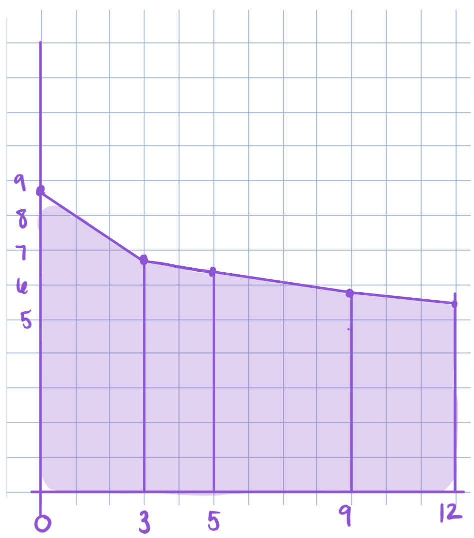


MRS

34
41.3
$\underline{}$
75.3 gallons

$120 + 75.3 = 195.3$ gallons

b



$\frac{1}{2}(3)(8.9+6.8) = 23.55$

$\frac{1}{2}(2)(6.8+6.4) = 13.2$

$\frac{1}{2}(4)(6.4+5.9) = 24.6$

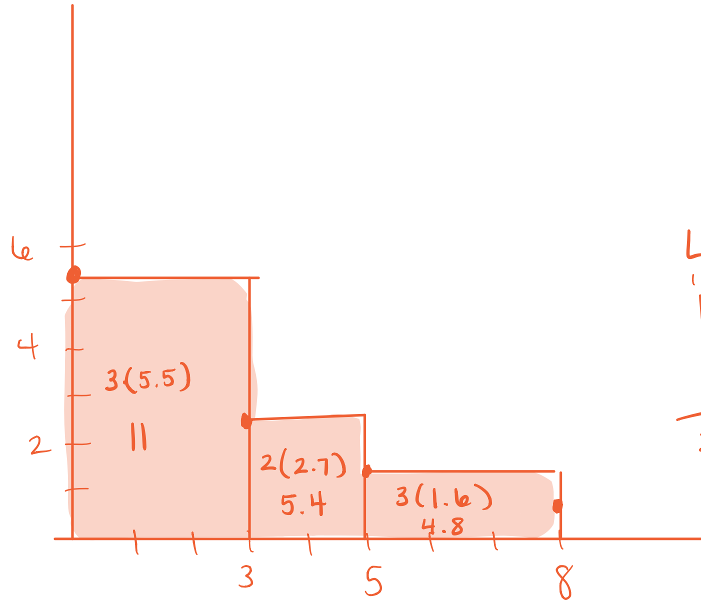
$\frac{1}{2}(3)(5.9+5.7) = 17.4$

78.75 gallons

$120 + 78.75 = 198.75$ gallons

3

a



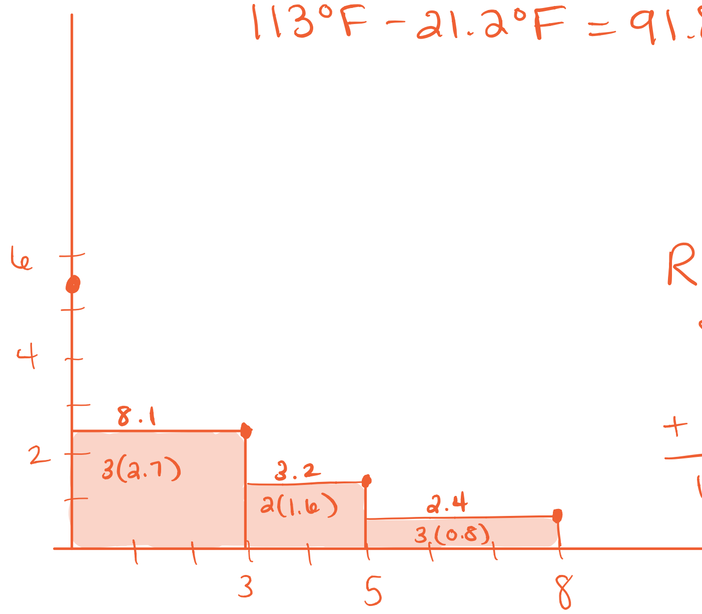
LRS

11
5.4
4.8

21.2° F

113°F − 21.2°F = 91.8°F

b



RRS

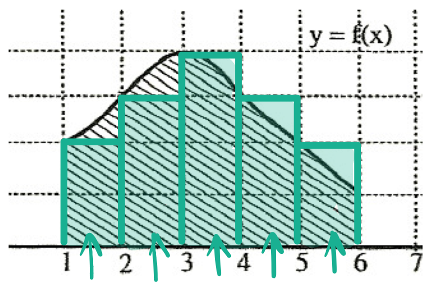
8.1
3.2
+ 2.4

13.7°F

113°F − 13.7°F = 99.3°F

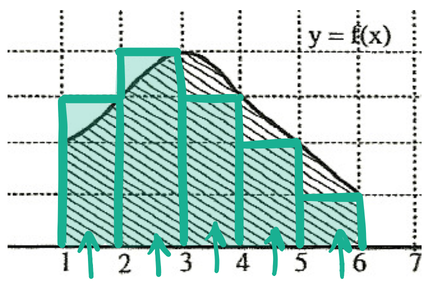
4

a



$2 + 3 + 4 + 3 + 2 = 14$

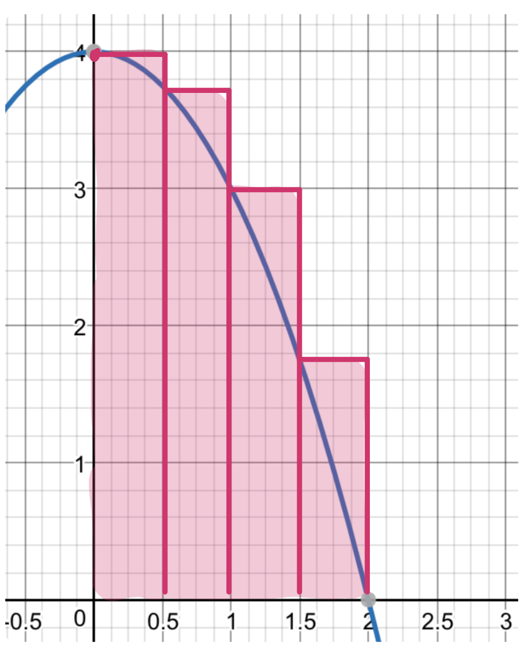
b



$3 \quad 4 \quad 3 \quad 2 \quad 1 = 13$

5

a



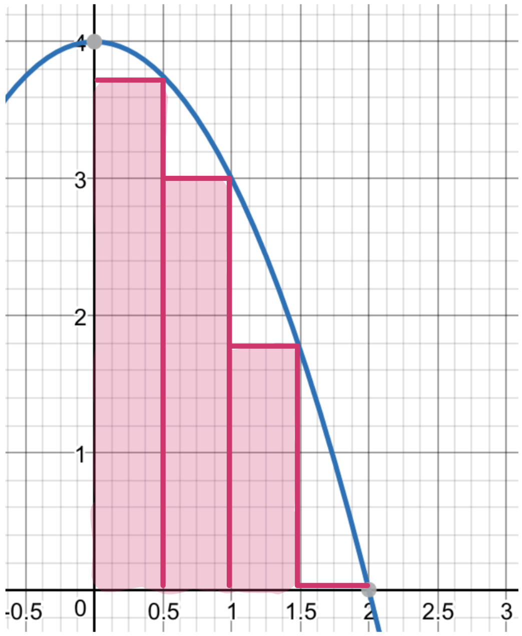
$\frac{1}{2}(4) = 2$

$\frac{1}{2}(3.75) = 1.875$

$\frac{1}{2}(3) = 1.5$

$\frac{1}{2}(1.75) = 0.875$

6.25



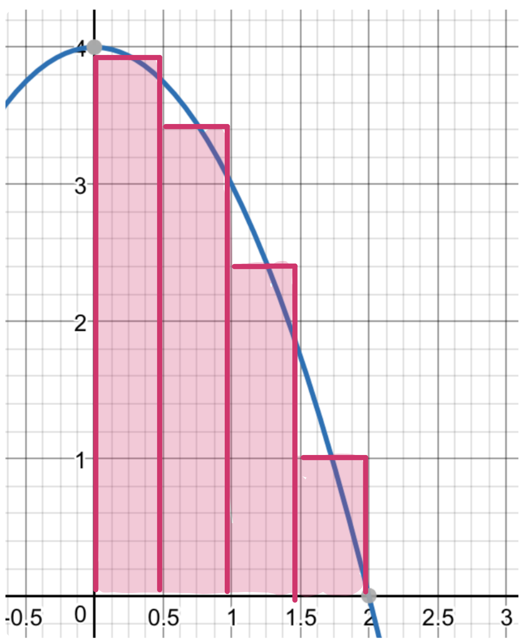
$\frac{1}{2}(3.75) = 1.875$

$\frac{1}{2}(3) = 1.5$

$\frac{1}{2}(1.75) = 0.875$

$\frac{1}{2}(0) = 0$

4.25



$\frac{1}{2}(4-.25^2) = 1.968$

$\frac{1}{2}(4-.75^2) = 1.718$

$\frac{1}{2}(4-1.25^2) = 1.218$

$\frac{1}{2}(4-1.75^2) = 0.468$

5.372

AB Calculus Name________________________________ ID: 1

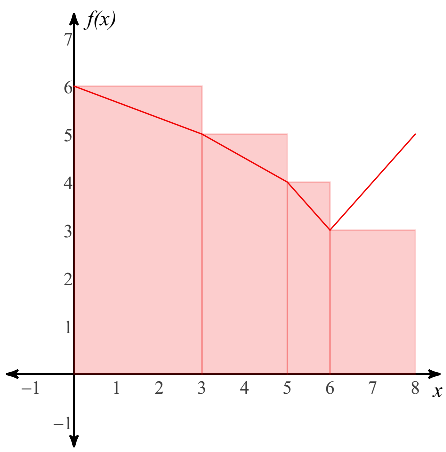
U7WS3 Reimann Sums & Trapezoid Rule

Date________________ Period____

For each problem, use a left-hand Riemann sum to approximate the integral based off of the values in the table. You may use the provided graph to sketch the function data and Riemann sums.

1) $\displaystyle\int_0^8 f(x)\,dx$

x	0	3	5	6	8
$f(x)$	6	5	4	3	5



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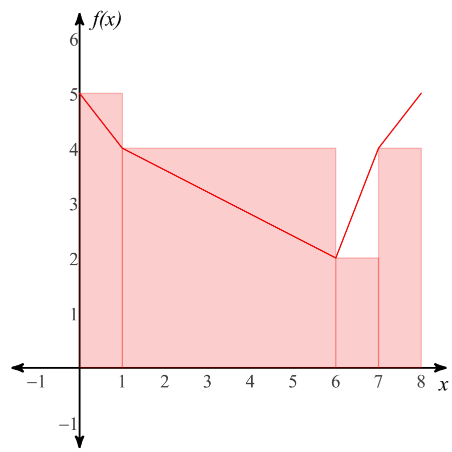
2) $\displaystyle\int_0^8 f(x)\,dx$

x	0	1	6	7	8
$f(x)$	5	4	2	4	5



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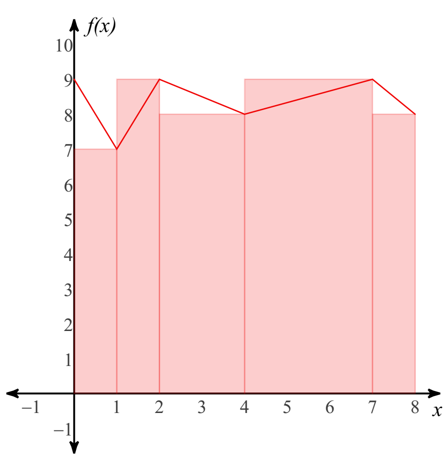
For each problem, use a right-hand Riemann sum to approximate the integral based off of the values in the table. You may use the provided graph to sketch the function data and Riemann sums.

3) $\displaystyle\int_0^8 f(x)\,dx$

x	0	1	2	4	7	8
$f(x)$	9	7	9	8	9	8



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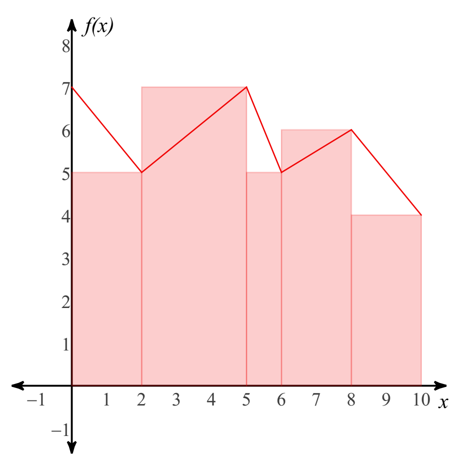
4) $\displaystyle\int_0^{10} f(x)\,dx$

x	0	2	5	6	8	10
$f(x)$	7	5	7	5	6	4



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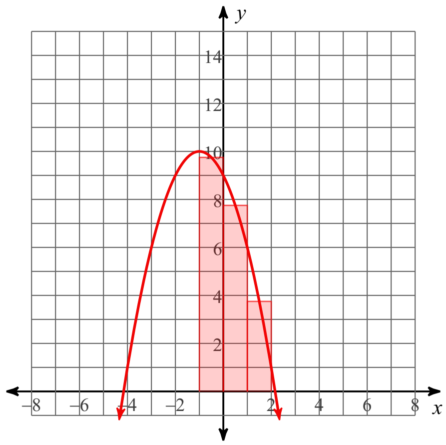
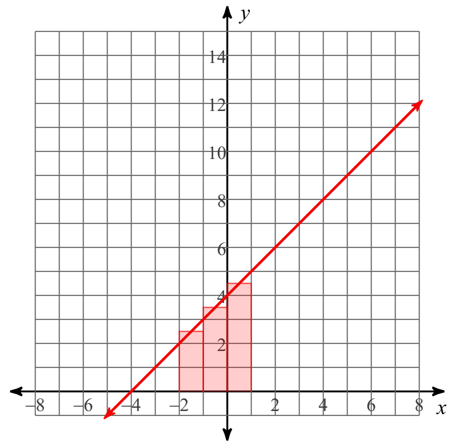
For each problem, approximate the area under the curve over the given interval using 3 midpoint rectangles. You may use the provided graph to sketch the curve and rectangles.

5) $y = -x^2 - 2x + 9$; $[-1, 2]$



$\frac{85}{4} = 21.25$

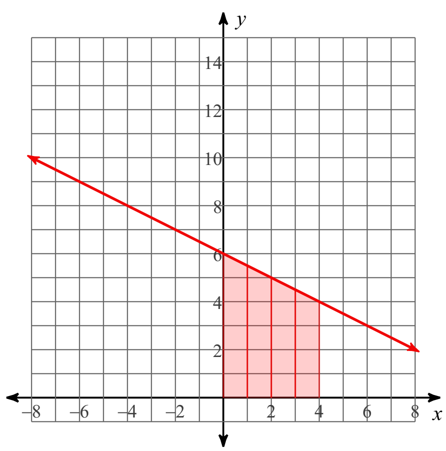
6) $y = x + 4$; $[-2, 1]$



$\frac{21}{2} = 10.5$

For each problem, approximate the area under the curve over the given interval using 4 trapezoids. You may use the provided graph to sketch the curve and trapezoids.

7) $y = -\frac{x}{2} + 6$; $[0, 4]$



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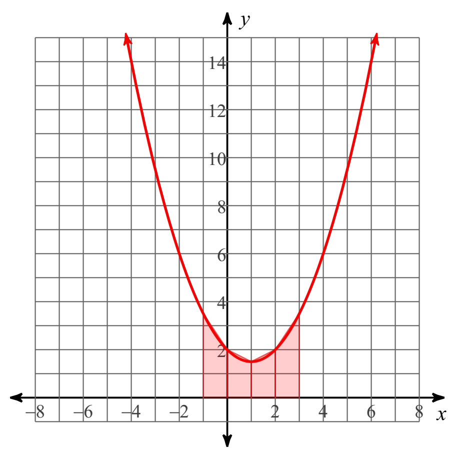
8) $y = \frac{x^2}{2} - x + 2$; $[-1, 3]$



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